A few tricks from different horizons of computer science

A mad warden

You are one of P recently arrested prisoners. The warden, a deranged computer scientist, makes the following announcement:

You may meet together today and plan a strategy, but after today you will be in isolated celles and have no communication with one another. I have set up a *switch room* which contains a light switch, which is either on or off. The switch is not connected to anything.

Every now and then, I will select one prisoner at random to enter the *switch room*. This prisoner may throw the switch, or may leave the switch unchanged. Nobody else will ever enter this room. Each prisoner will visit the switch room arbitrarily often. More precisely, for any N, eventually each of you will visit the switch room at least N times.

At any time, any of you may declare *"we have all visited the switch room at least once"*. If the claim is correct, I will set you free. If the claim is incorrect, I will feed all of you to the crocodiles.

Choose wisely! Devise a winning strategy when you know that the initial state of the switch is off. Device one if you do not know the initial state of the switch.

The blind croupier

A blind croupier is presented with 4 cards organized on a square. His goal is to have all four cards on the same side, either all side up, or all side down, which a faithful friend will notice him as soon as it happens.

However, a wicked player tries to prevent him from achieving its work: each time the croupier return a subset of all cards, the player rotates the square of any angle between $\{0, \pi/2, \pi, 3\pi/2\}$ without noticing the croupier of its act.

Can you devise a winning strategy for the croupier to win under a fixed amount of moves no matter what the player does? What is the shortest strategy?

Kakeya's pin

Consider a pin of length 1 and without gridth in the plan. Your goal is to rotate it without lifting nor bending it. Can you perform so in a shape whose area is smaller than one of a circle of diameter 1? What is the smallest area you can find allowing it?